The Application of Propensity Score Methods

A Powerful Tool For Observational Studies

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Education

- 2023 — 至今 Institute of Statistical Science, Academia Sinica. Postdoc.
- 2017 — 2023 Institute of Epidemiology and Preventive Medicine, National Taiwan University. Ph.D.
  Taiwan Cancer Registry, Taipei, Taiwan. R.A.
- 2012 — 2016 Department of Statistics, National Cheng Kung University. B.S.

Research Interest

- Biostatistics and Epidemiology
- Statistical Genetics
- Machine Learning
- Smart Healthcare

Selected Publications

- **Jhuang, J. R.**, Chiang, C. J., Su, S. Y., Yang, Y. W., & Lee, W. C. (2019). Reduction in the Incidence of Urological Cancers after the Ban on Chinese Herbal Products Containing Aristolochic Acid: An Interrupted Time-Series Analysis. Scientific reports, 9(1), 19860. https://doi.org/10.1038/s41598-019-56394-y
- **Jhuang, J. R.**, Lee, W. C., & Chan, C. C. (2020). A randomized, double-blind water taste test to evaluate the equivalence of taste between tap water and filtered water in the Taipei metropolis. Scientific reports, 10(1), 13387. https://doi.org/10.1038/s41598-020-70272-y
- **Jhuang, J. R.**, Su, S. Y., Chiang, C. J., Yang, Y. W., Lin, L. J., Hsu, T. H., & Lee, W. C. (2022). Forecast of peak attainment and imminent decline after 2017 of oral cancer incidence in men in Taiwan. Scientific reports, 12(1), 5726. https://doi.org/10.1038/s41598-022-09736-2
- **Jhuang, J. R.**, Chiu, P. C., Hsieh, T. C., Chen, C. H., Pu, Y. S., & Lee, W. C. (2023). Latency period of aristolochic acid-induced upper urinary tract urothelial carcinoma. Frontiers in public health, 11, 1072864. https://doi.org/10.3389/fpubh.2023.1072864
- **Jhuang, J.R.**, Lee, C.H., Chiang, C.J., Chen, C.J., & Lee, W.C. (2024). Reduced Burden of Arsenic-Related Cancers After Water Mitigation in Taiwan. Environment International, 185, 108542. https://doi.org/10.1016/j.envint.2024.108542

Abstract

Introduction

- Real examples
- Causal inference
- Propensity Score

Propensity Score Methods

- Matching
- Inverse probability weighting
- Stratification
- Regression

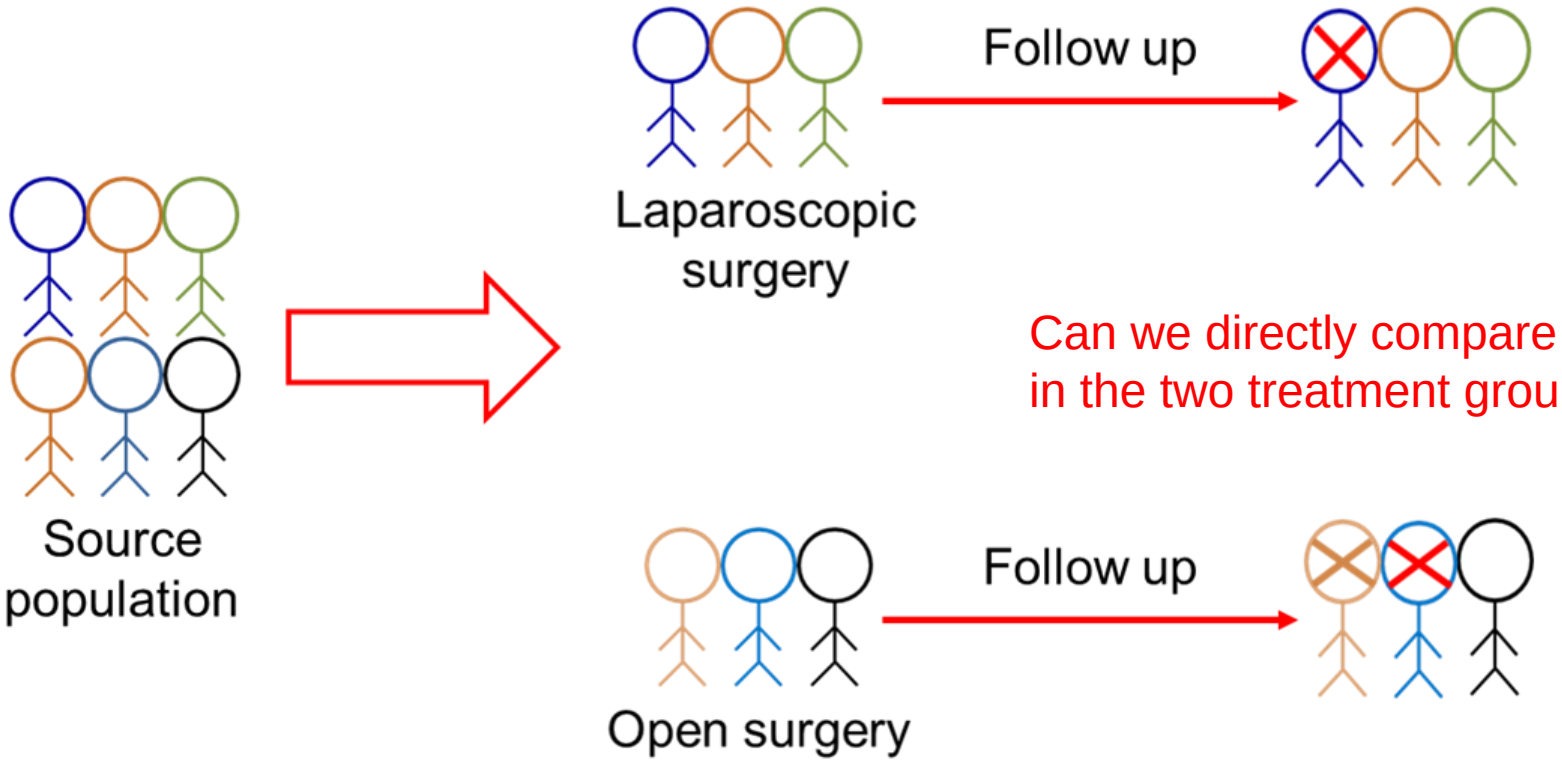
Case Analysis

- Phototherapy for newborn jaundice

Summary and Conclusion

- Limitation and special topics

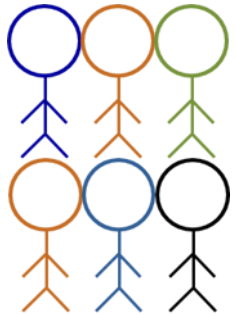
Laparoscopic surgery examples



Can we directly compare the outcomes in the two treatment groups?

Causal Inference

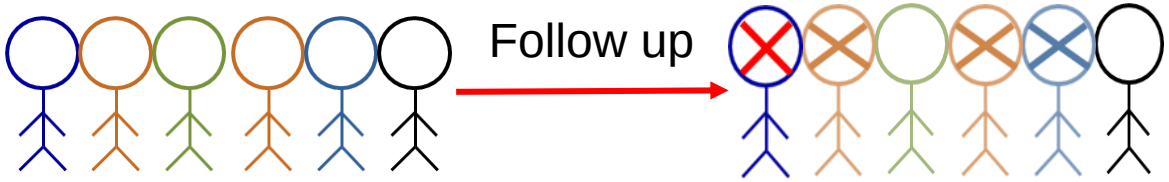
- Counterfactual framework
  - What if ?



Source population

Laparoscopic surgery

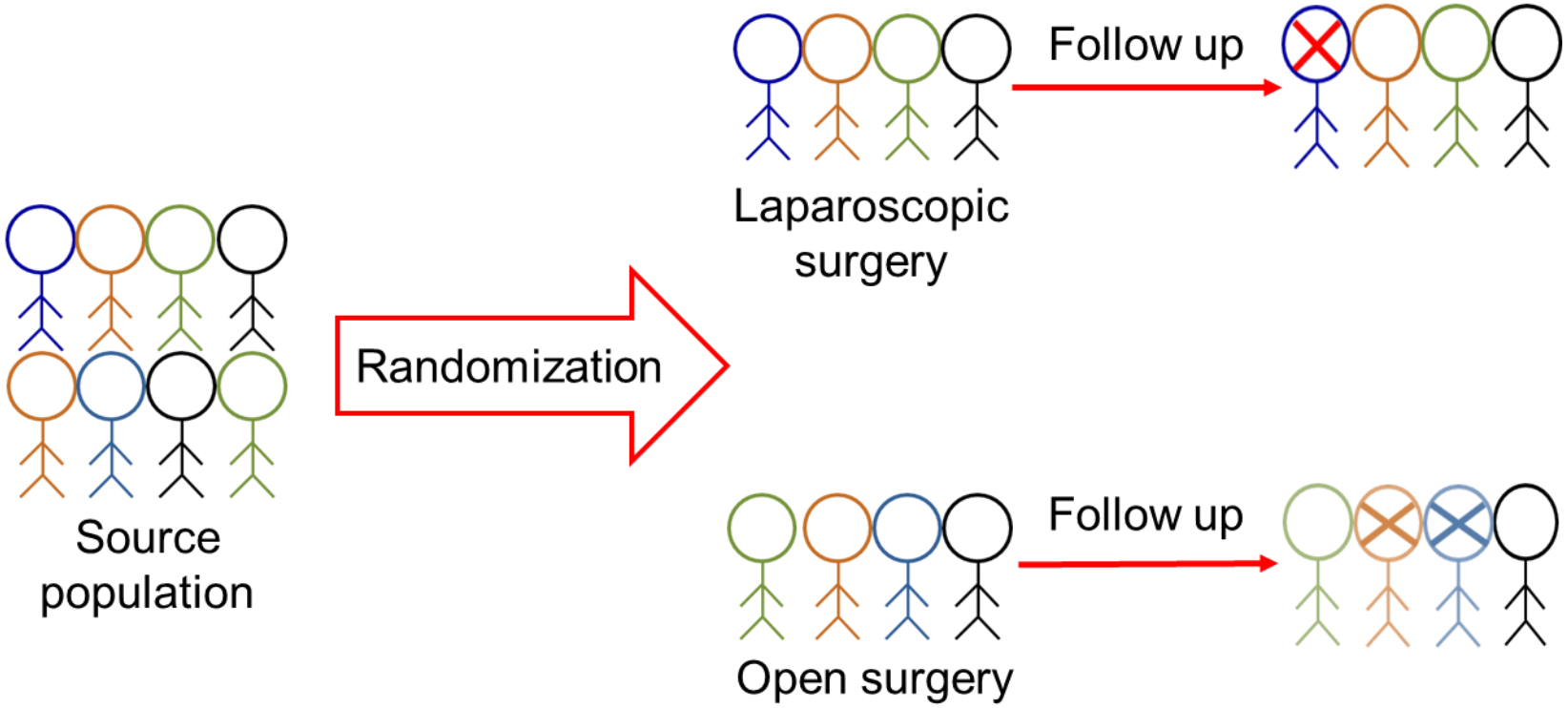
In this hypothetical setting,
we can directly compare the outcomes
In the two treatment groups.



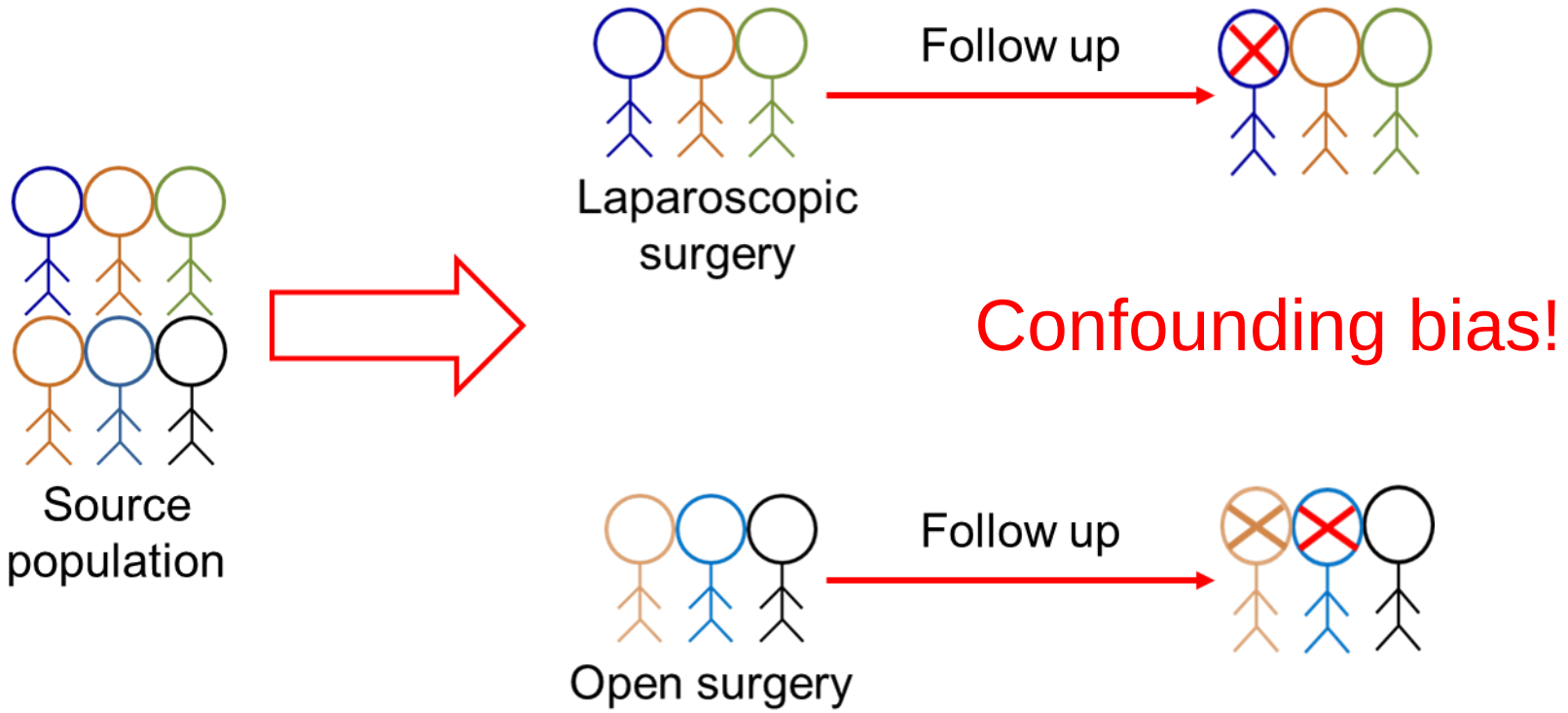
Open surgery

How do we create the parallel universes in the real world ?

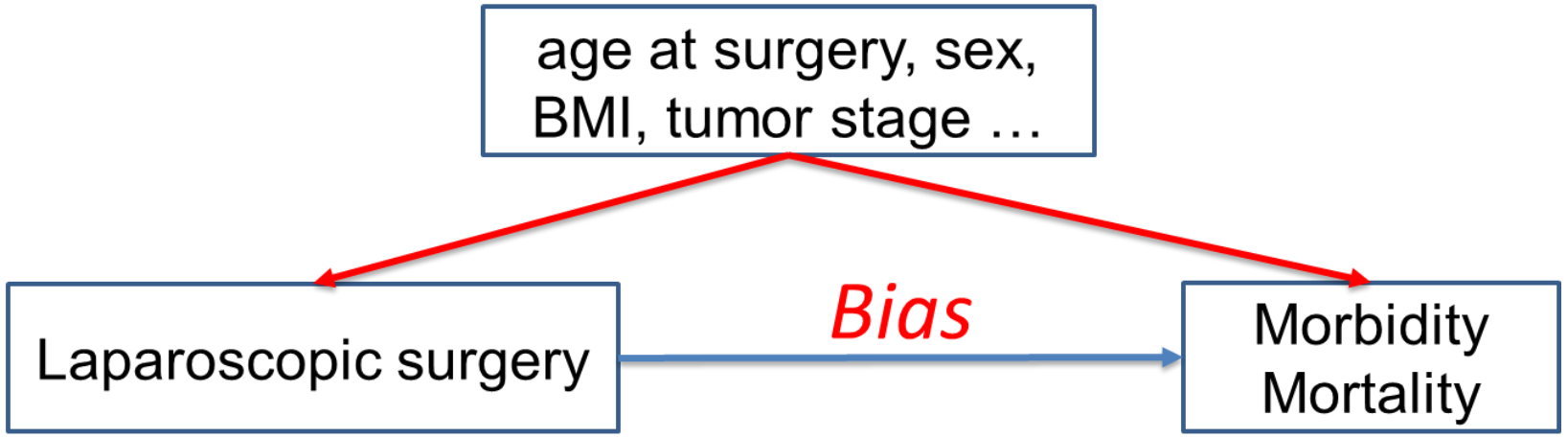
Randomization



Causal Inference in Cohort Study



Confounding bias

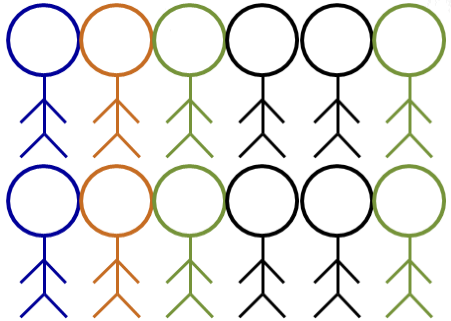


Matching | Stratification | Regression

Matching

Laparoscopic surgery

Open surgery



Tumor stage is a well-known confounder in cancer studies.

It is hard to adjust a lot of confounders by using matching.

Stratification and Regression

Stratification

1. Residual confounding
2. Number of strata increase dramatically

Regression

1. Collinearity problem
2. Assumption of linear association

JSTOR
https://www.jstor.org › stable

The Central Role of the Propensity Score in Observational ...

由 PR Rosenbaum 著作 · 1983 · 被引用 37839 次 — The propensity score is the conditional probability of assignment to a particular treatment given a vector of observed covariates. Both…

Can we analyze cohort data as if it is from a randomized trial?

Efficient and Safe Method for Splenic Flexure Mobilization in Laparoscopic Left Hemicolectomy: A Propensity Score-weighted Cohort Study

Yu-Jen Hsu, Yih-Jong Chern, **Jing-Rong Jhuang**, Wen-Sy Tsai, Jy-Ming Chiang, Hsin-Yuan Hung, Tzong-Yun Tsai, Jeng-Fu You. Surg Laparosc Endosc Percutan Tech. (2020)

Decreasing Postoperative Pulmonary Complication Following Laparoscopic Surgery in Elderly Individuals with Colorectal Cancer: A Competing Risk Analysis in a Propensity Score–Weighted Cohort Study

Yih-Jong Chern, Jeng-Fu You, Ching-Chung Cheng, **Jing-Rong Jhuang**, Chien-Yuh Yeh, Pao-Shiu Hsieh, Wen-Sy Tsai, Chun-Kai Liao, Yu-Jen Hsu. Cancers. (2022)

Comparison of Laparoscopic and Open Surgery for Colorectal Malignancy in Obese Patients: A Propensity Score-Weighted Cohort Study

Yu-Jen Hsu, Jeng-Fu You, **Jing-Rong Jhuang**, Yen-Lin Yu, Yih-Jong Chern. Int J Surg. (2022)

	Overall survival				Disease-free survival			
	Before PSW		After PSW		Before PSW		After PSW	
	HR [95% CI]	*p*-value	HR [95% CI]	*p*-value	HR [95% CI]	*p*-value	HR [95% CI]	*p*-value
General population								
Laparoscopy group	0.74 [0.43-1.27]	-	0.98 [0.69-1.38]	0.85	0.69 [0.44-1.07]	-	0.89 [0.67-1.18]	0.69
Open group	1.00	-	1.00	-	1.00	-	1.00	-
The elderly population								
Laparoscopy group	0.94 [0.78-1.13]	-	1.07 [0.95-1.21]	0.50	0.94 [0.79-1.12]	-	1.08 [0.97-1.21]	0.39
Open group	1.00	-	1.00	-	1.00	-	1.00	-
The obese population								
Laparoscopy group	1.32 [0.89-1.95]	0.17	1.43 [1.10-1.86]	<0.01	1.22 [0.87-1.72]	0.26	1.39 [1.09-1.75]	<0.01
Open group	1.00	-	1.00	-	1.00	-	1.00	-

Association between prognostic performance and disability level among oral cancer patients: a hospital-based study in Taiwan

Chih-Chin Lai#, **Jing-Rong Jhuang#**, Kuo-Liong Chien, Jiu-Jenq Lin, Hsin-Hui Peng, Shin-Liang Pan. J Formos Med Assoc. (2024) – in preparation.

	Univariate analysis			Multivariate analysis		
Variables	**HR**	**95% CI**	***p*-value**	**HR**	**95% CI**	***p*-value**
WHODAS score						
≥60	**1.59**	**(1.05-2.40)**	**0.03**	**1.45**	**(0.95-2.21)**	**0.08**
<60	**1.00**	**-**	**-**	**1.00**	**-**	**-**
Age						
≥65 years	1.01	(0.56-1.81)	0.98	1.21	(0.62-2.35)	0.58
<65 years	1.00	-	-	1.00	-	-
Sex						
Male	1.43	(0.45-4.51)	0.55	0.92	(0.26-3.25)	0.90
Female	1.00	-	-	1.00	-	-
Smoker						
Yes	1.62	(0.78-3.35)	0.19	1.44	(0.63-3.30)	0.39
No	1.00	-	-	1.00	-	-
Alcohol consumption						
Yes	1.22	(0.77-1.94)	0.40	1.02	(0.62-1.66)	0.94
No	1.00	-	-	1.00	-	-
Betel quid chewing						
Yes	1.92	(1.07-3.46)	0.03	1.57	(0.82-3.00)	0.17
No	1.00	-	-	1.00	-	-
Tumor stage						
Stage IV	1.77	(109.-2.85)	0.02	1.41	(0.86-2.29)	0.17
stage I-III	1.00	-	-	1.00	-	-
Tumor site						
Tongue	1.12	(0.73-1.71)	0.60	1.13	(0.74-1.75)	0.57
Others	1.00	-	-	1.00	-	-
Treatment						
Surgery and CCRT	3.47	(1.74-6.91)	<0.01	3.07	(1.51-6.23)	<0.01
Others	1.00	-	-	1.00	-	-
Charlson Comorbidity Index	0.99	(0.85-1.17)	0.94	1.02	(0.85-1.23)	0.81

	Propensity score matching	
	HR [95% CI]	*p*-value
WHODAS score ≥ 60	1.89 [1.15-3.11]	0.01
WHODAS score < 60	1.00	-

Potential Outcomes Model

	$Y^{(1)}$	$Y^{(0)}$
Laparoscopic surgery (X = 1)	Observable $E[Y^{(1)} \mid X = 1]$	Counterfactual $E[Y^{(0)} \mid X = 1]$
Open surgery (X = 0)	Counterfactual $E[Y^{(1)} \mid X = 0]$	Observable $E[Y^{(0)} \mid X = 0]$

Data in the Real World

	$Y^{(1)}$	$Y^{(0)}$	$\mathbf{E}(\mathbf{Y}^{(1)}) - \mathbf{E}(\mathbf{Y}^{(0)})$
Patient 1	15	?	?
Patient 2	13	?	?
Patient 3	?	8	?
Patient 4	?	4	?

Naïve estimate = 4

Data in Hypothetical World

	$Y^{(1)}$	$Y^{(0)}$	$\delta = \mathbf{E}(\mathbf{Y}^{(1)}) - \mathbf{E}(\mathbf{Y}^{(0)})$
Patient 1	15	10	5
Patient 2	13	8	5
Patient 3	13	8	5
Patient 4	9	4	5

ATT = ATU; ATE = 5

Data in Hypothetical World

	$Y^{(1)}$	$Y^{(0)}$	$\delta = \mathbf{E}(\mathbf{Y}^{(1)}) - \mathbf{E}(\mathbf{Y}^{(0)})$
Patient 1	15	10	5
Patient 2	13	8	5
Patient 3	11	8	3
Patient 4	7	4	3

ATT ≠ ATU; ATE = 4

Causal effect under counterfactual framework

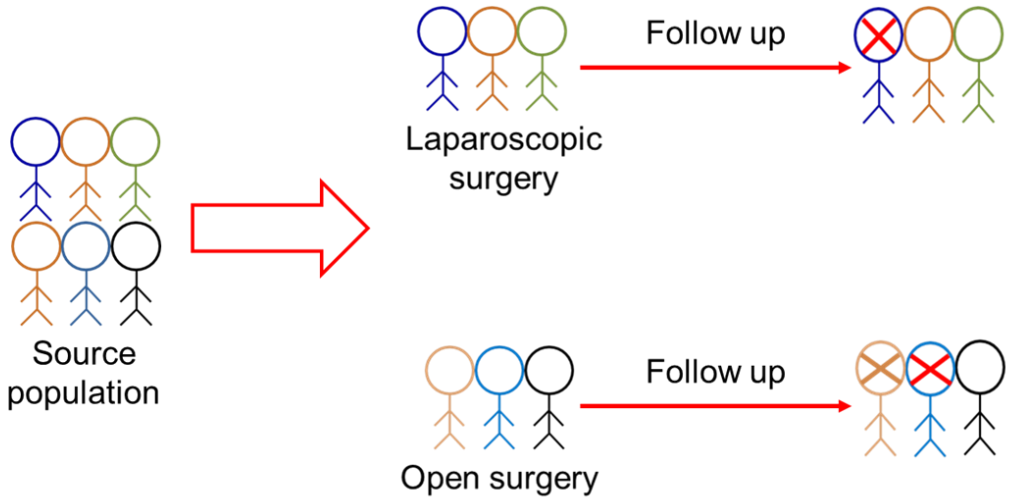
- ATE（Average Treatment Effect）:
  $\delta_{\mathrm{ATE}} = E[Y^{(1)}] - E[Y^{(0)}] = \pi\delta_{\mathrm{ATT}} + (1-\pi)\delta_{\mathrm{ATU}}$
- ATT（Average Treatment Effect on the Treated）:
  $\delta_{\mathrm{ATT}} = E[Y^{(1)} | X = 1] - E[Y^{(0)} | X = 1]$
- ATU（Average Treatment Effect on the Untreated）:
  $\delta_{\mathrm{ATU}} = E[Y^{(1)} | X = 0] - E[Y^{(0)} | X = 0]$

However, there is a trouble!!!

$$\delta_{obs} = E[Y^{(1)} | X = 1] - E[Y^{(0)} | X = 0]$$

Causal effect under counterfactual framework

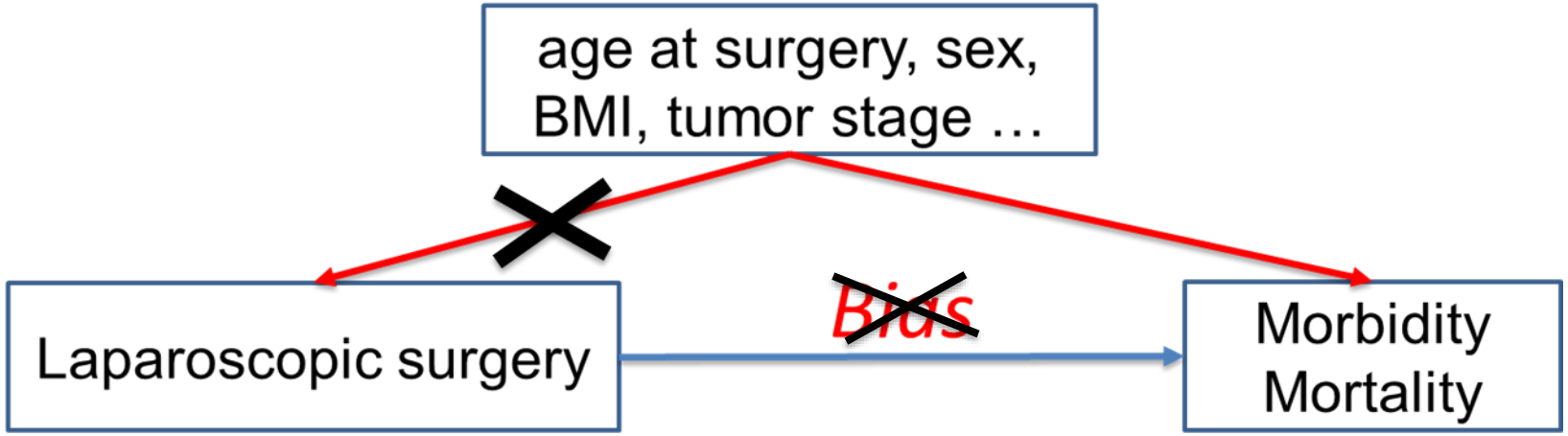
Naïve Estimate	$E[Y^{(1)}\|X = 1] - E[Y^{(0)}\|X = 0]$
= average causal effect	$= E(\delta = Y^{(1)} - Y^{(0)})$
+ baseline bias	$+ \{E(Y^{(0)}\|X = 1) - E(Y^{(0)}\|X = 0)\}$
+ differential effect bias	$+ \{E(\delta\|X = 1) - E(\delta\|X = 0)\}\ (1-\pi)$



Are these possible?

$$\delta_{\mathbf{ATE}} = \delta_{\mathbf{obs}}$$
$$\delta_{\mathbf{ATT}} = \delta_{\mathbf{obs}}$$

Propensity Score



$$e(Z)=\Pr[\ \text{laparoscopic}=1 \mid Z=(\text{age},\text{sex},\text{BMI},\text{stage})\ ]$$

$$Z \perp X=\text{laparoscopic} \mid e(Z)$$

This important property of PS can break the association between treatment and measured confounders

Strongly Ignorable Treatment Assignment (Exchangeability)

$(Y^{(1)}, Y^{(0)}) \perp X=\text{laparoscopic} \mid e(Z)$

- This assumption means *if there is no other unmeasured confounder*, then *the parallel universes can be created* given those patients with similar PS.

$$
\begin{aligned}
\delta_{obs} &= E\left[Y^{(1)} \middle| X=1\right] - E\left[Y^{(0)} \middle| X=0\right] \\
&= E\left[E\left[Y^{(1)} \middle| X=1, e(Z)\right] - E\left[Y^{(0)} \middle| X=0, e(Z)\right]\right] \\
&= E\left[E\left[Y^{(1)} \middle| e(Z)\right] - E\left[Y^{(0)} \middle| e(Z)\right]\right] \\
&= E\left[Y^{(1)}\right] - E\left[Y^{(0)}\right] = \delta_{ATE}
\end{aligned}
$$

Estimation of Propensity score

Let $P_i = \Pr(\text{Laparoscopic surgery}_i | \text{Age}_i, \text{BMI}_i, \text{Stage}_i)$

$$\log\left(\frac{P_i}{1 - P_i}\right) = \beta_0 + \beta_1 \text{age}_i + \beta_2 \text{BMI}_i + \beta_3 \text{stage}_i$$

$$\widehat{\mathbf{P}}_\mathbf{i} = \frac{\exp(\widehat{\boldsymbol{\beta}}_\mathbf{0} + \widehat{\boldsymbol{\beta}}_\mathbf{1}\mathbf{age}_\mathbf{i} + \widehat{\boldsymbol{\beta}}_\mathbf{2}\mathbf{BMI}_\mathbf{i} + \widehat{\boldsymbol{\beta}}_\mathbf{3}\mathbf{stage}_\mathbf{i})}{\mathbf{1} + \exp(\widehat{\boldsymbol{\beta}}_\mathbf{0} + \widehat{\boldsymbol{\beta}}_\mathbf{1}\mathbf{age}_\mathbf{i} + \widehat{\boldsymbol{\beta}}_\mathbf{2}\mathbf{BMI}_\mathbf{i} + \widehat{\boldsymbol{\beta}}_\mathbf{3}\mathbf{stage}_\mathbf{i})}$$

Estimate the PS using SAS or R

```
PROC LOGISTIC data=laparoscopic descending;
class laparoscopic age BMI stage;
model laparoscopic = age BMI stage;
output out=propensity_scores pred = prob;
run;

glm(laparoscopic ~ age + BMI + stage,
    data = laparoscopic,
    family = binomial)
```

Check the Balance between Groups

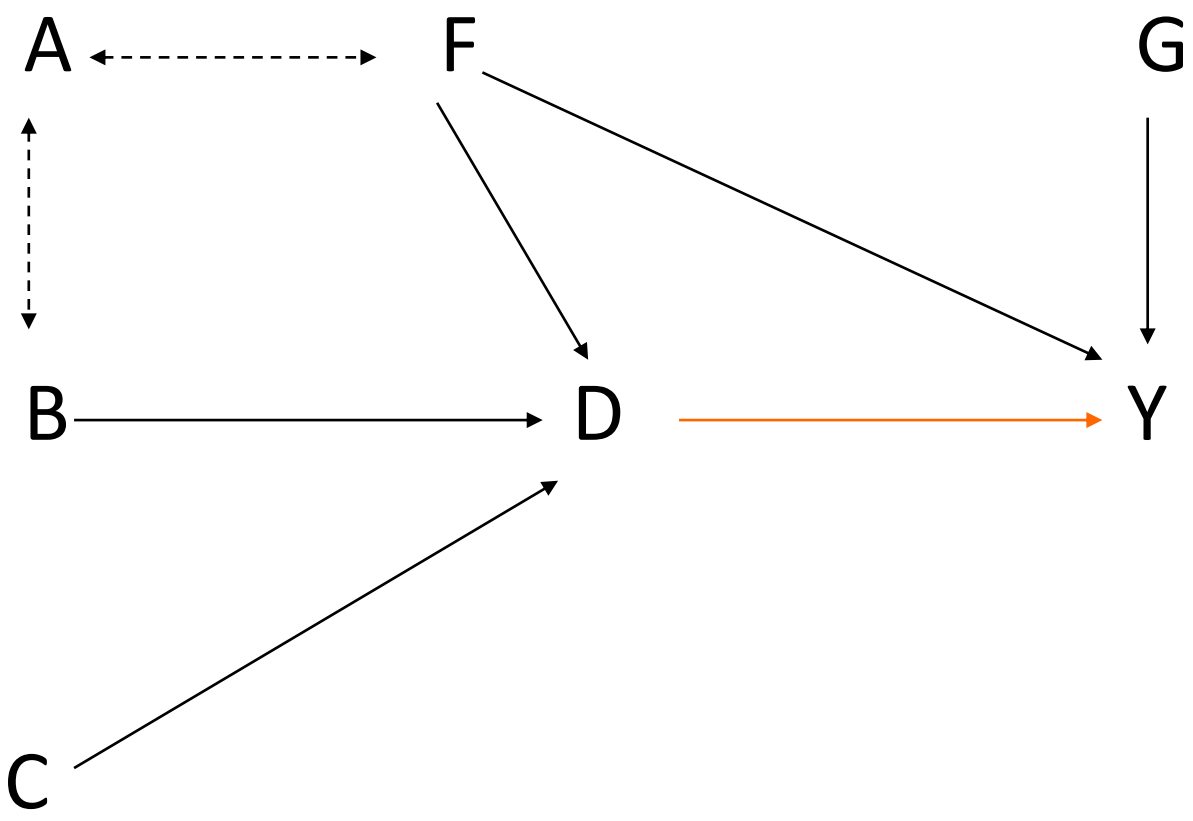
Table 1. Baseline characteristics of patients treated with open surgery vs. laparoscopy before and after propensity score weighting.

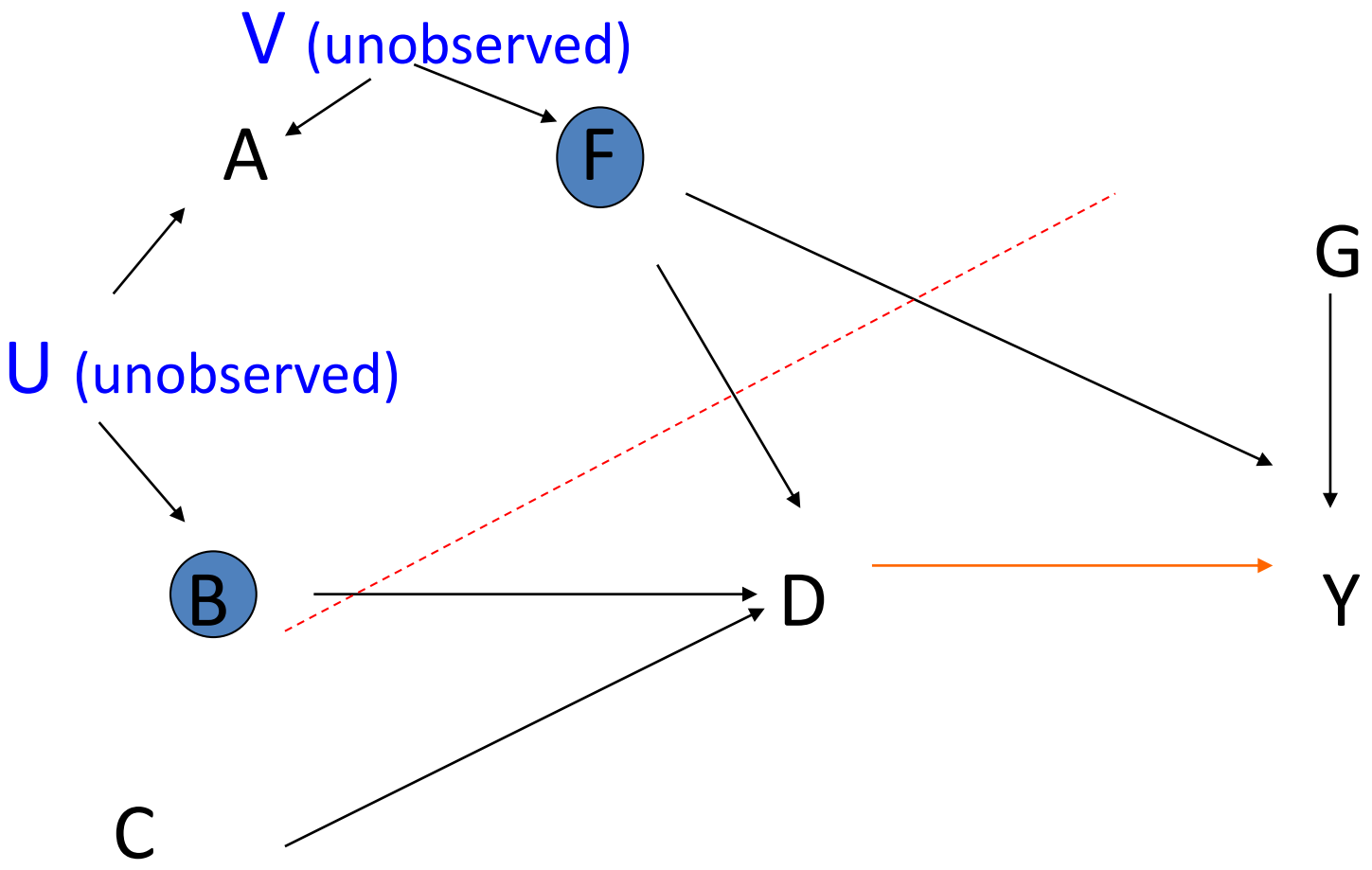
Variables	Open Group	Laparoscopic Group	*p*-Value	
	n = 846	*n* = 504	Before	After
Age, No. (%)			0.85	0.95
75–80 years	415 (49.05)	250 (49.60)		
≥80 years	431 (50.95)	254 (50.40)		
Gender, No. (%)			0.73	0.92
Female	378 (44.68)	230 (45.63)		
Male	468 (55.32)	274 (54.37)		
BMI, No. (%)			<0.01	0.75
<25	596 (70.45)	315 (62.50)		
≥25	250 (29.55)	189 (37.50)		
Previous abdominal operation, No. (%)				
Appendectomy	97 (11.47)	54 (10.71)	0.67	0.02
Cholecystectomy	59 (6.97)	36 (7.14)	0.91	0.01
Hysterectomy	50 (5.91)	20 (3.97)	0.12	0.09
Oophorectomy	14 (1.65)	6 (1.19)	0.49	0.03
Colon-rectal operation	44 (5.20)	12 (2.38)	0.01	0.10
total	584 (69.03)	336 (66.67)	0.37	<0.01
Operation, No. (%)				
Hartmann resection	38 (4.49)	16 (3.17)	0.23	0.59
Abdomino-peritoneal	18 (2.13)	14 (2.78)	0.45	0.08
Anterior resection	473 (55.91)	295 (58.53)	0.35	0.92
Left hemicolectomy	55 (6.50)	35 (6.94)	0.75	0.17
Right hemicolectomy	225 (26.60)	139 (27.58)	0.69	0.48
Segmental resection	18 (2.13)	2 (0.40)	0.01	<0.01
Subtotal colectomy	19 (2.25)	3 (0.60)	0.02	<0.01
Comorbidity, No. (%)				
Hypertension	479 (56.62)	327 (64.88)	<0.01	0.89
Cardiac disease	151 (17.85)	90 (17.86)	1.00	0.99
Cerebrovascular accident	70 (8.27)	32 (6.35)	0.20	0.59
Asthma	39 (4.61)	23 (4.56)	0.97	0.91
Diabetes mellitus	214 (25.30)	131 (25.99)	0.78	0.98
Liver Cirrhosis	96 (11.35)	56 (11.11)	0.89	0.98
others	277 (32.74)	186 (36.90)	0.12	0.81
Carcinoembryonic antigen, No. (%)			<0.01	0.65
<5 ng/mL	537 (63.48)	365 (72.42)		
≥5 ng/mL	309 (36.52)	139 (27.58)		
Hemoglobin, No. (%)			1.00	1.00
<10 mg/mL	225 (26.60)	134 (26.59)		
≥10 mg/mL	621 (73.40)	370 (73.41)		
Albumin, No. (%)			<0.01	0.78
<3.5 mg/dL	200 (23.64)	82 (16.27)		
≥3.5 mg/dL	646 (76.36)	422 (83.73)		
Total bilirubin, No. (%)			0.78	0.77
≤1.3	831 (98.23)	494 (98.02)		
>1.3	15 (1.77)	10 (1.98)		
Creatinine, No. (%)			0.55	0.75
≤1.3	709 (83.81)	416 (82.54)		
>1.3	137 (16.19)	88 (17.46)		
Tumor stage, No. (%)			<0.01	0.96
1	111 (13.12)	100 (19.84)		
2	369 (43.62)	224 (44.44)		
3	366 (43.26)	180 (35.71)		
Histological type, No. (%)			0.36	0.60
Adenocarcinoma	794 (93.85)	479 (95.04)		

Table 1. *Cont.*

Variables	Open Group	Laparoscopic Group	*p*-Value	
	n = 846	*n* = 504	Before	After
Mucinous adenocarcinoma & Signet ring cell	52 (6.15)	25 (4.96)		
Histology grade, No. (%)			0.01	0.80
Poorly differentiated	85 (10.05)	30 (5.95)		
Moderately differentiated	682 (80.61)	412 (81.75)		
Well differentiated	79 (9.34)	62 (12.30)		
Retrieved lymph node (+) number, No. (%)			0.01	0.18
<12	47 (5.56)	13 (2.58)		
≥12	799 (94.44)	491 (97.42)		
Tumor site, No. (%)			0.52	0.91
Left side colon	325 (38.42)	182 (36.11)		
Anus & rectum	266 (31.44)	173 (34.33)		
Right side colon	255 (30.14)	149 (29.56)		
Tumor size, No. (%)			<0.01	0.99
<4 cm	422 (49.88)	297 (58.93)		
≥4 cm	424 (50.12)	207 (41.07)		
Stomy type, No. (%)			0.07	0.96
No	671 (79.31)	425 (84.33)		
Diverting stomy	103 (12.17)	46 (9.13)		
End stomy	72 (8.51)	33 (6.55)		

Which covariates should be controlled ?




V (unobserved)
A
F
G
U (unobserved)
B
D
Y
C

Major Applications of PS

1. Propensity score matching
2. Inverse probability weighting
3. Stratification
4. Regression

Propensity Score Matching

Selection
- Select potential confounders

PS
- Estimate propensity scores using logistic regression

Matching
- Greedy nearest neighborhood matching

Quality Checks
- Check the balance of covariates by univariate analysis

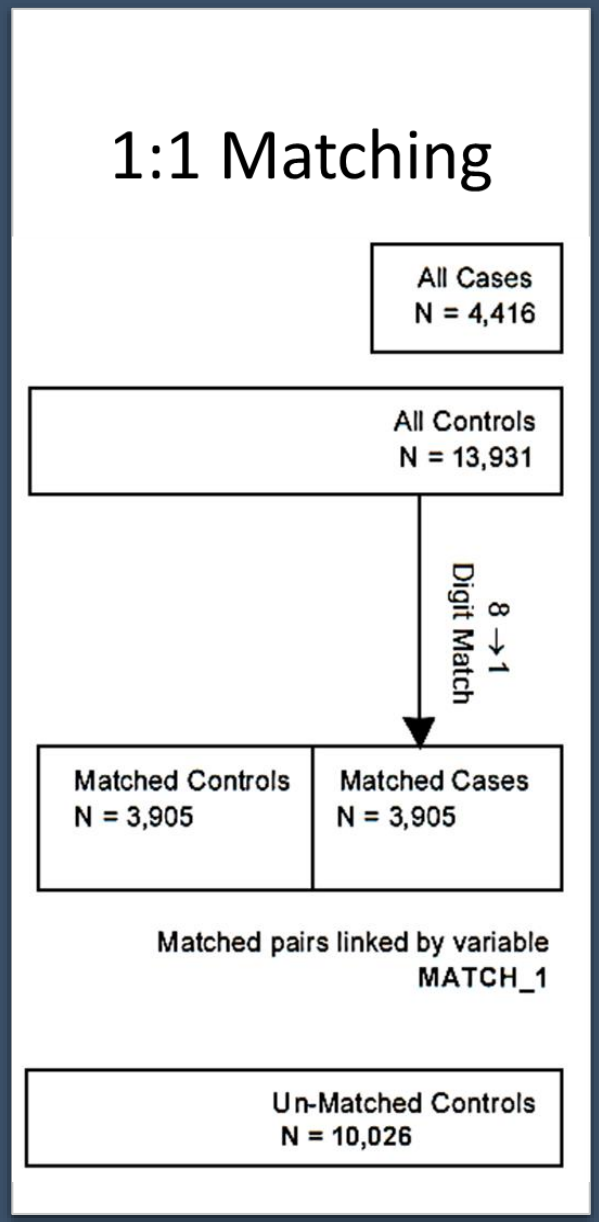
Estimate
- Estimate the *average treatment effect on the treated (ATT)*

Greedy Nearest Neighbor Matching

- Radius (Caliper)
  - An acceptable distance of the two PS to be matched
  - Rosenbaum & Rubin（1985）suggested that

$$|P_i - P_j| < \varepsilon \text{ ; } \varepsilon \leqq .25\ \sigma_p$$

σ_p is the SD of the PS.



Matching

- Algorithm
  - Greedy Nearest Neighbor Matching
    - Most popular
  - Optimal Matching
    - large sample size
  - Full Matching
    - small sample size
- Distance choose
  - Euclidean distance
  - Mahalanobis distance
  - Kernel function
- Other settings
  - Radius (calipers)

Effect Estimation

Let $P_i = \Pr(\text{Death}_i | \text{Laparoscopic}_i)$

$$\log\left(\frac{P_i}{1 - P_i}\right) = \beta_0 + \beta_1 \text{Laparoscopic}_i$$

$$\widehat{P}_i = \frac{\exp(\widehat{\beta}_0 + \widehat{\beta}_1 \text{Laparoscopic}_i)}{1 + \exp(\widehat{\beta}_0 + \widehat{\beta}_1 \text{Laparoscopic}_i)}$$

NOTE: Because matched data is paired data, *conditional logistic regression* should be used.

Software for PSM

- **SPSS:** Thoemmes, F. (2012). (http://sourceforge.net/projects/psmspss/files/)

- **SAS:** Macro for GREEDY Matching (https://support.sas.com/resources/papers/proceedings17/0812-2017.pdf)

- **R Packages:** MatchIt; Matching

- **Stata:** psmatch2、nnmatch、teffects (STATA 13)

Propensity Score Matching using SAS

```
%MACRO OneToManyMTCH (
Lib=work, /* Library Name */
Dataset=Propensity_scores, /* Data set of all patients */
Depend=Laparoscopic, /* Dependent variable that indicates
treated or non-treated，Code 1 for treated, 0 for non-treated*/
SiteN=hospital, /* Site/Hospital ID */
PatientN=id, /* Patient ID */
matches=matches, /* Output data set of matched pairs */
NoContrls=1); /* Number of controls to match to each case */

/*Conditional logistic regression*/
PROC LOGISTIC DATA=matches descending;
CLASS death laparoscopic(ref='no');
MODEL death = laparoscopic;
STRATA matched_id;
RUN;
```

Inverse Probability Weighting

Selection

- Select confounders that may affect the assignment of the treatment

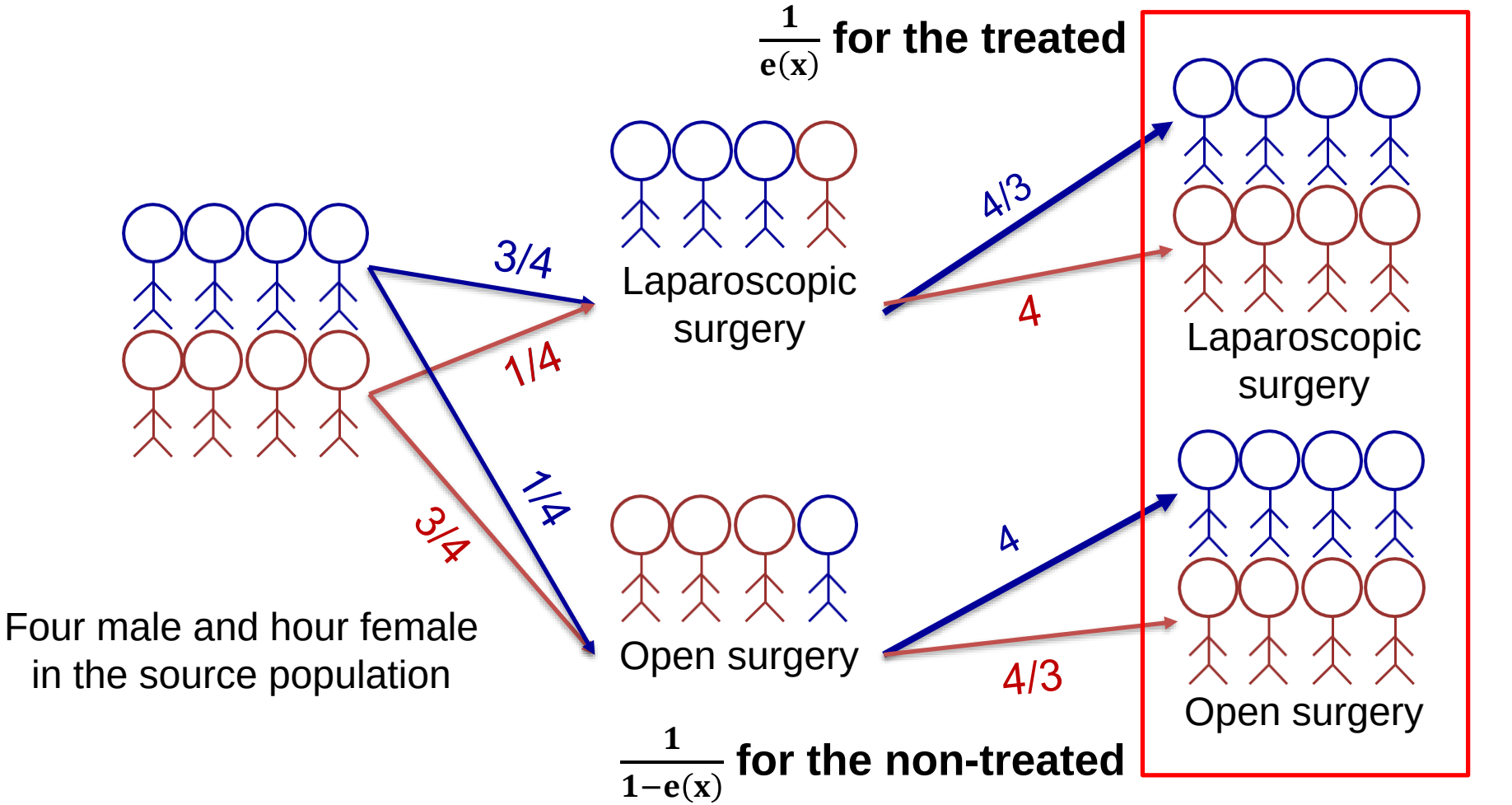
PS

- Estimate propensity scores using logistic regression

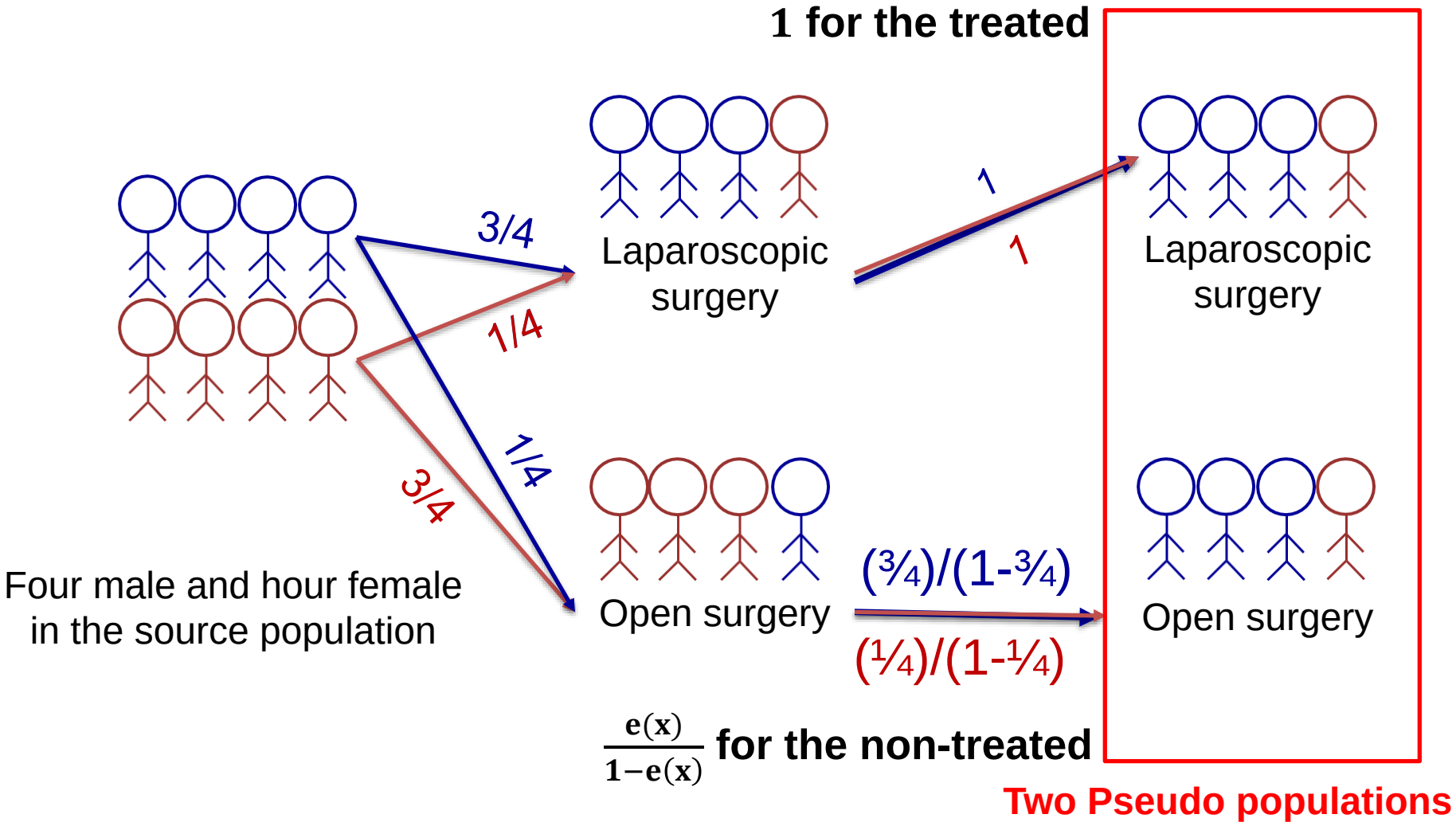
Estimate

- Fit the regression model and include the PS as weight

Inverse Probability Weighting for ATE



Inverse Probability Weighting for ATT



IPW using SAS

```
*Computing inverse weights;
DATA IPW;
set propensity_scores;
if laparoscopic='1' then do
ipw_ATE=1/prob; ipw_ATT=1;
end;
if laparoscopic='0' then do
ipw_ATE=1/(1-prob); ipw_ATT=prob/(1-prob);
end;
run;

PROC LOGISTIC data=IPW descending;
class death laparoscopic(ref='0');
model death = laparoscopic;
weight ipw_ATE; /*ipw_ATT*/
run;
```

Stratification & Regression

Selection

- Select confounders that may affect the assignment of the treatment

PS

- Estimate propensity scores using logistic regression

Estimate

1. Create several PS groups for stratification
2. Include the PS groups in the regression

Case Analysis

Phototherapy for newborn jaundice

Follow-up more than 20000 of newborns

https://drive.google.com/file/d/12HuJUImgPe5s0qZBeT8AX7ZPcOCcBAer/view?usp=sharing

- Original Data

birth_wt	bwcat	gest_age	hospital	id	male	over_thresh	phototherapy	qual_TSB	year
3.599999905	3.5-3.9 kg	39	1&4	1	no	no	no	-1 to <0	2000
4.5	>= 4.0 kg	39	1&4	2	yes	no	no	0 to <1	2003
5.099999905	>= 4.0 kg	38	1&4	3	yes	no	yes	0 to <1	2001
2.5	2.5-2.9 kg	36	1&4	4	yes	no	no	1 to <2	2001
3.5	3.5-3.9 kg	40	1&4	5	yes	no	no	-2 to <-1	1998
3.400000095	3.0-3.4 kg	38	1&4	6	yes	no	no	-3 to <-2	2001
2.900000095	2.5-2.9 kg	37	1&4	7	yes	no	no	1 to <2	1999
3.900000095	3.5-3.9 kg	39	1&4	8	no	no	yes	0 to <1	2004
3.799999952	3.5-3.9 kg	38	1&4	9	no	no	no	-2 to <-1	2003

- Propensity_scores Data

gest_age_cat	age_days	birth_wt	bwcat	gest_age	hospital	id	male	over_thresh	phototherapy	qual_TSB	year	回應值	估計機率
<41	48 to <72h	3.6	3.5-3.9 kg	39	1&4	1	no	no	0	-1 to <0	2000	1	0.117443
<41	72 to <96h	4.5	>= 4.0 kg	39	1&4	2	yes	no	0	0 to <1	2003	1	0.10945
<41	72 to <96h	5.1	>= 4.0 kg	38	1&4	3	yes	no	1	0 to <1	2001	1	0.089466
<41	24 to <48h	2.5	2.5-2.9 kg	36	1&4	4	yes	no	0	1 to <2	2001	1	0.494203
<41	>= 96h	3.5	3.5-3.9 kg	40	1&4	5	yes	no	0	-2 to <-1	1998	1	0.037297
<41	>= 96h	3.4	3.0-3.4 kg	38	1&4	6	yes	no	0	-3 to <-2	2001	1	0.038853
<41	<24 h	2.9	2.5-2.9 kg	37	1&4	7	yes	no	0	1 to <2	1999	1	0.642581
<41	72 to <96h	3.9	3.5-3.9 kg	39	1&4	8	no	no	1	0 to <1	2004	1	0.089398
<41	>= 96h	3.8	3.5-3.9 kg	38	1&4	9	no	no	0	-2 to <-1	2003	1	0.047616

Estimate Propensity score

- Use *stepwise regression* to select potential confounders.

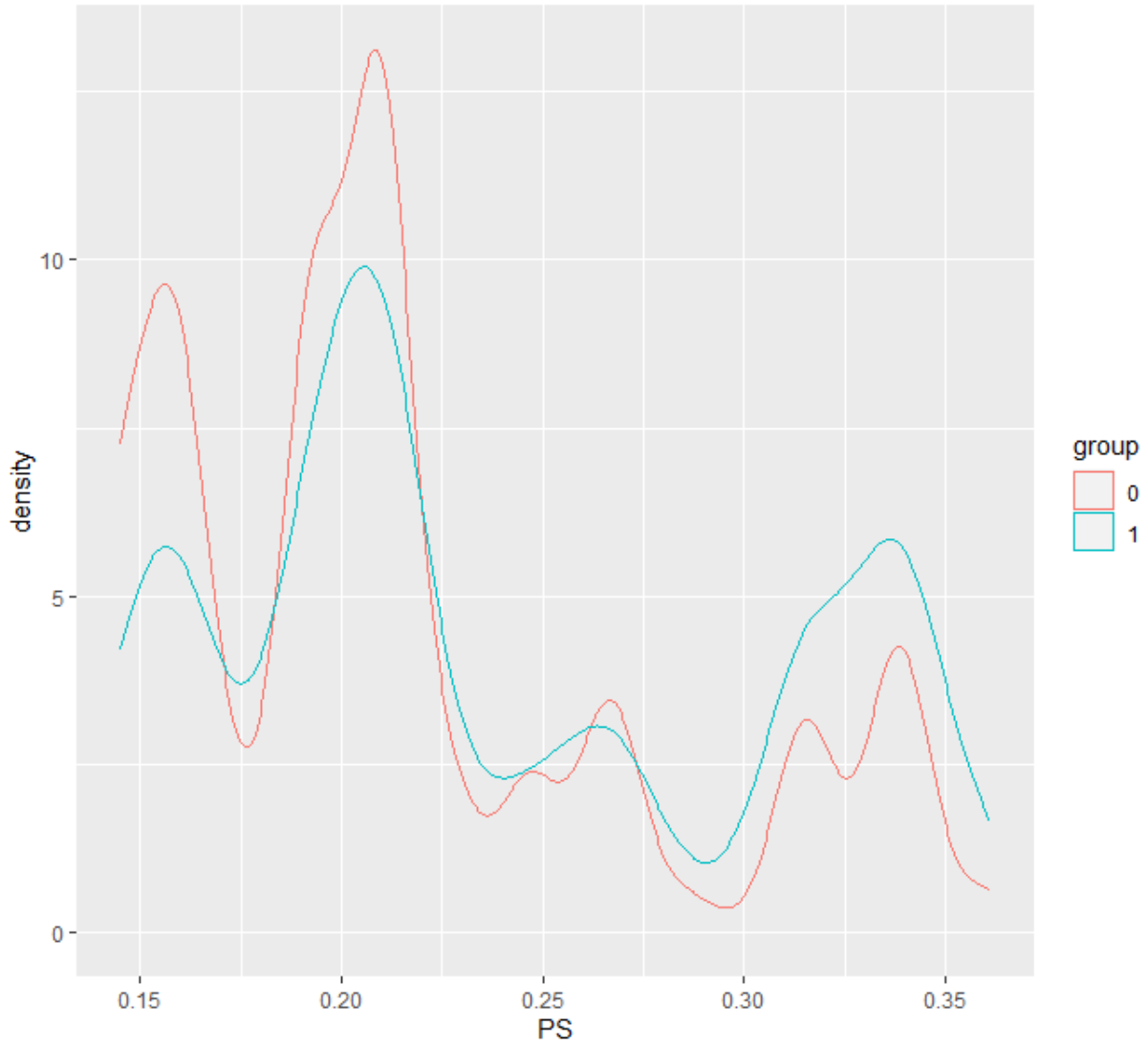
```
PROC LOGISTIC data=photo descending;
Class phototherapy male bwcat age_days hospital gest_age_cat
qual_TSB year hospital;
Model phototherapy = male bwcat age_days gest_age_cat qual_TSB year
hospital / selection=stepwise ;
output out=propensity_scores pred = prob;
run;
```

- Final model:

$$\log\left(\frac{P_i}{1-P_i}\right) = \beta_0 + \beta_1 \text{gest\_age\_cat}_i + \beta_2 \text{male}_i + \beta_3 \text{TSB}_i + \beta_4 \text{bwcat}_i + \beta_5 \text{year}_i$$

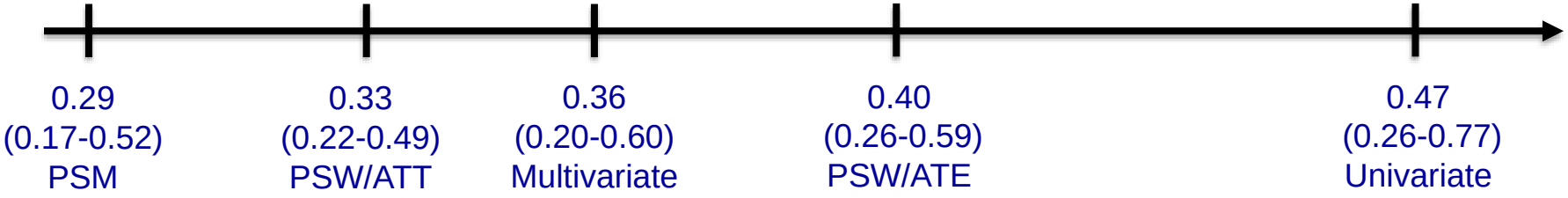
$$\widehat{P}_i = \frac{\exp(\widehat{\beta}_0 + \widehat{\beta}_1 \text{gest\_age\_cat}_i + \widehat{\beta}_2 \text{male}_i + \widehat{\beta}_3 \text{TSB}_i + \widehat{\beta}_4 \text{bwcat}_i + \widehat{\beta}_5 \text{year}_i)}{1 + \exp(\widehat{\beta}_0 + \widehat{\beta}_1 \text{gest\_age\_cat}_i + \widehat{\beta}_2 \text{male}_i + \widehat{\beta}_3 \text{TSB}_i + \widehat{\beta}_4 \text{bwcat}_i + \widehat{\beta}_5 \text{year}_i)}$$

Quality Check

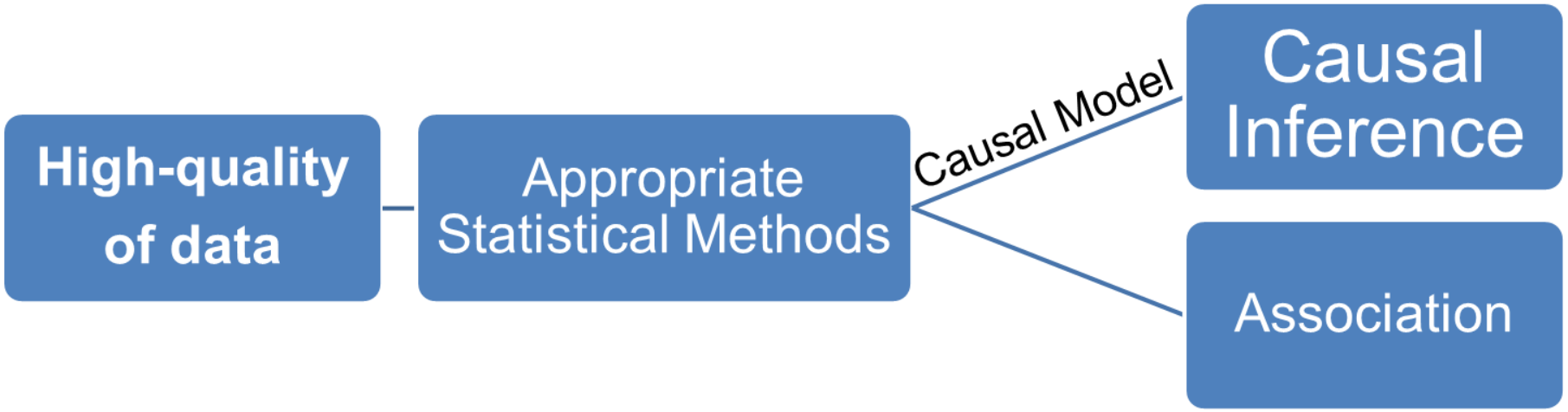


Odds Ratio Estimation

The lower the OR, the stronger the effect for the phototherapy.



Summary and Conclusion


High-quality of data
Appropriate Statistical Methods
Causal Model
Causal Inference
Association

Summary

Limitation

Counterfactual model may be incorrect.

Large sample size is required.

PS cannot make unobserved confounders balance between groups (Stukel et al. *JAMA*, 2007)

Special topics

- Estimation of Propensity Score with Machine Learning Model [Lee et al. (2009)]
- Improvement of the balancing score with Doubly robust estimation [Funk et al. (2011)]
- Cross-sectional study [Mason et al. (2019)]
- Case-control study [Roger et al. (2007)]
  - Matching may cause ***artificial bias***.
- Cohort study
  - Survival analysis [Peter C. Austin (2014)]
  - Mediation analysis [Jo et al. (2011)]
- Longitudinal study
  - Random effects or GEE
  - Time-varying effects [Wijn et al. (2021)]

Thanks for your listening！

Q&A

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TAIWAN